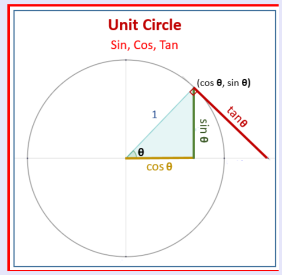
Trigonometry

Lecture 23



Feb 19-8:47 AM

Verify

$$\csc x\,[\csc x + \sin(-x)] = \cot^2 x$$

1) Recall $\sin(-\alpha) = -\sin\alpha$

$$\csc x = \frac{1}{\sin x}$$

$$\csc x\,(\csc x + \sin(-x)) = \frac{1}{\sin x}\left(\frac{1}{\sin x} - \sin x\right)$$

Recall $\csc x = \frac{1}{\sin x}$

$$= \frac{1}{\sin^2 x} - 1$$

$$= \csc^2 x - 1$$

$1 + \cot^2 x = \csc^2 x$

$$= \cancel{1} + \cot^2 x \cancel{-1}$$

$$= \boxed{\cot^2 x}$$

Oct 8-10:26 AM

Verify

$$\frac{1}{1-\sin x} - \frac{1}{1+\sin x} = 2\sec x \tan x$$

$$\frac{1\,(1+\sin x)}{(1-\sin x)(1+\sin x)} - \frac{1\,(1-\sin x)}{(1+\sin x)(1-\sin x)} =$$

$$\frac{\cancel{1}+\sin x \;\cancel{-1}+\sin x}{(1-\sin x)(1+\sin x)} = \frac{2\sin x}{1-\sin^2 x}$$

$$= \frac{2\sin x}{\cos^2 x}$$

$$= 2\cdot\boxed{\frac{\sin x}{\cos x}}\cdot\boxed{\frac{1}{\cos x}}$$

$$= 2\cdot\tan x\cdot\sec x \checkmark$$

Oct 8-10:31 AM

Verify

$$\frac{1+\sin x}{1-\sin x} - \frac{1-\sin x}{1+\sin x} = 4\tan x \sec x$$

$$\frac{(1+\sin x)(1+\sin x)}{(1-\sin x)(1+\sin x)} - \frac{(1-\sin x)(1-\sin x)}{(1+\sin x)(1-\sin x)} =$$

$$\frac{(1+\sin x)(1+\sin x) - (1-\sin x)(1-\sin x)}{(1+\sin x)(1-\sin x)} =$$

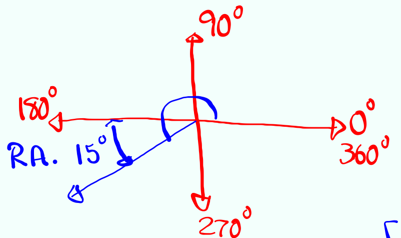
$$\frac{1+\sin x+\sin x+\sin^2 x - (1-\sin x-\sin x+\sin^2 x)}{1-\sin^2 x} =$$

$$= \frac{\cancel{1}+2\sin x+\cancel{\sin^2 x}\;\cancel{-1}+2\sin x-\cancel{\sin^2 x}}{\cos^2 x} = \frac{4\sin x}{\cos^2 x}$$

$$= 4\cdot\frac{\sin x}{\cos x}\cdot\frac{1}{\cos x} = \boxed{4\tan x\cdot\sec x}$$

Oct 8-10:36 AM

Find exact Value of $\cos 195^\circ$



$15^\circ = 45^\circ - 30^\circ$

$= -\cos 15^\circ$

$A - B$

$= -\cos(45^\circ - 30^\circ)$

$= -\left[\cos 45^\circ \cos 30^\circ + \sin 45^\circ \sin 30^\circ\right]$

$= -\left[\frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2} \cdot \frac{1}{2}\right]$

$= -\left(\frac{\sqrt{6}}{4} + \frac{\sqrt{2}}{4}\right)$

$= \boxed{-\frac{\sqrt{6} + \sqrt{2}}{4}}$

Oct 8-10:45 AM

Find exact Value of

$$\frac{\tan \frac{\pi}{18} + \tan \frac{\pi}{9}}{1 - \tan \frac{\pi}{18} \cdot \tan \frac{\pi}{9}}$$

$$\frac{\tan A + \tan B}{1 - \tan A \tan B}$$

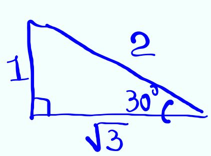
$= \tan(A + B)$

$= \tan\left(\frac{\pi}{18} + \frac{\pi}{9}\right)$

$= \tan\left(\frac{\pi}{18} + \frac{2\pi}{18}\right) = \tan \frac{\cancel{3}\pi}{\cancel{18}_6}$

$= \tan \frac{\pi}{6} = \tan 30^\circ$

$\sin 30^\circ = \frac{1}{2}$



$= \frac{1}{\sqrt{3}} = \boxed{\frac{\sqrt{3}}{3}}$

Oct 8-10:50 AM

Verify

$$\cos\left(\frac{\pi}{2} - x\right) = \sin x$$

$$\cos\left(\frac{\pi}{2} - x\right) = \cos A \cos B + \sin A \sin B$$

$\cos(A - B)$

$$= \cancel{\cos\frac{\pi}{2}}^{\,0} \cos x + \cancel{\sin\frac{\pi}{2}}^{\,1} \sin x$$

$$= 0 \cdot \cos x + 1 \cdot \sin x$$

$$= \boxed{\sin x}$$

Oct 8-10:55 AM

Simplify

$$\cos(x + 30^\circ) + \sin(x - 60^\circ) =$$

$\cos(A + B)$ $\quad$ $\sin(A - B)$

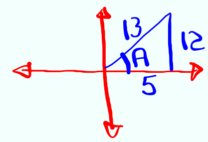
$$\cos x \cos 30^\circ - \sin x \sin 30^\circ + \sin x \cos 60^\circ - \cos x \cdot \sin 60^\circ$$

$$= \cancel{\frac{\sqrt{3}}{2}\cos x} - \cancel{\frac{1}{2}\sin x} + \cancel{\frac{1}{2}\sin x} - \cancel{\frac{\sqrt{3}}{2}\cos x}$$

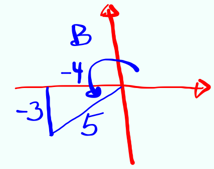
$$= \boxed{0}$$

Oct 8-10:58 AM

$\sin A = \frac{12}{13}$, A is in QI



$\tan B = \frac{3}{4}$, B is is QIII



Find $\tan(A-B) = \dfrac{\tan A - \tan B}{1 + \tan A \tan B}$

$$= \frac{\frac{12}{5} - \frac{3}{4}}{1 + \frac{12}{5}\cdot\frac{3}{4}}$$

$$= \frac{\overset{4}{\cancel{20}}\cdot\frac{12}{\cancel{5}} - \overset{5}{\cancel{20}}\cdot\frac{3}{\cancel{4}}}{20\cdot 1 + \cancel{20}\cdot\frac{12}{\cancel{5}}\cdot\frac{3}{\cancel{4}}} = \frac{48-15}{20+36} = \boxed{\frac{33}{56}}$$

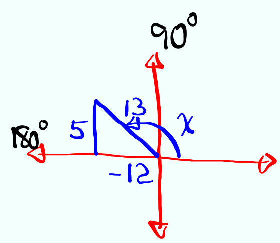
LCD = 20

Oct 8-11:03 AM

$\sin x = \frac{5}{13}$, x is in QII

$90^\circ < x < 180^\circ$

$180^\circ < 2x < 360^\circ$



Find $\sin 2x$ & $\cos 2x$.

Recall $\sin 2x = 2\sin x \cos x$

$\cos 2x = \cos^2 x - \sin^2 x$

$\sin 2x = 2\sin x \cos x$

$= 2\cdot\frac{5}{13}\cdot\frac{-12}{13}$

$= \boxed{\frac{-120}{169}}$

$\cos 2x = \cos^2 x - \sin^2 x$

$= \left(\frac{-12}{13}\right)^2 - \left(\frac{5}{13}\right)^2 =$

$= \frac{144}{169} - \frac{25}{169} = \boxed{\frac{119}{169}}$

$2x$ is in QIII or $\boxed{\text{QIV}}$

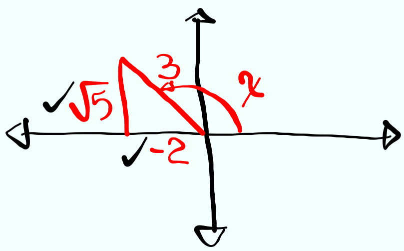
$2x$ is in QI or $\boxed{\text{QIV}}$

$2x$ is QIV

$270^\circ < 2x < 360^\circ$

Oct 8-11:09 AM

$\cos x = -\frac{2}{3}$, x is in QII , find $\tan 2x$.



$$\tan 2x = \frac{2\tan x}{1 - \tan^2 x}$$

$$= \frac{\cancel{2} \cdot \frac{\sqrt{5}}{\cancel{-2}}}{1 - \left(\frac{\sqrt{5}}{-2}\right)^2}$$

$$= \frac{-\sqrt{5}}{1 - \frac{5}{4}} = \frac{\cancel{-}4\sqrt{5}}{\underset{\cancel{-1}}{4-5}} = \boxed{4\sqrt{5}}$$

Oct 8-11:17 AM

$$\cos 2x = \cos^2 x - \sin^2 x$$

Replace $\sin^2 x$ with $1 - \cos^2 x$

$$\cos 2x = \cos^2 x - (1 - \cos^2 x)$$

$$= \cos^2 x - 1 + \cos^2 x$$

$$= 2\cos^2 x - 1$$

$$\boxed{\cos 2x = 2\cos^2 x - 1}$$

If we replace $\cos^2 x$ with $1 - \sin^2 x$

$$\cos 2x = \cos^2 x - \sin^2 x$$

$$= 1 - \sin^2 x - \sin^2 x$$

$$= 1 - 2\sin^2 x$$

$$\boxed{\cos 2x = 1 - 2\sin^2 x}$$

Oct 8-11:22 AM

Double-angle Formulas:

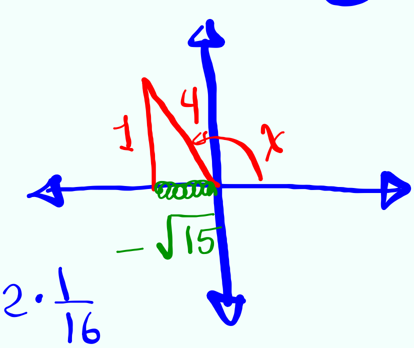
$$\boxed{\begin{aligned} \sin 2x &= 2\sin x\cos x \\ \cos 2x &= \cos^2 x - \sin^2 x \\ &= 2\cos^2 x - 1 \\ &= 1 - 2\sin^2 x \\ \tan 2x &= \frac{2\tan x}{1-\tan^2 x} \end{aligned}}$$

Oct 8-11:26 AM

$\csc x = 4 \rightarrow \sin x = \frac{1}{4}$ $\quad \sin x > 0$ I, II

$\tan x < 0$ $\quad \tan x < 0$ II, IV

Find $\cos 2x$

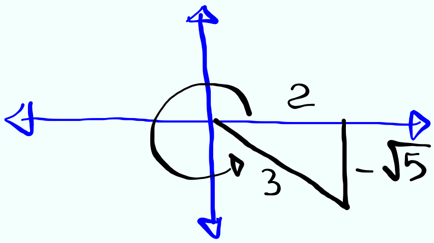


$$\cos 2x = 1 - 2\sin^2 x$$
$$= 1 - 2\cdot\left(\frac{1}{4}\right)^2 = 1 - 2\cdot\frac{1}{16}$$
$$= 1 - \frac{1}{8} = \boxed{\frac{7}{8}}$$

Oct 8-11:28 AM

$\sec x = \frac{3}{2}$, $\sin x < 0$ Find $\sin 2x$.

$\cos x = \frac{2}{3}$ $\cos x > 0 \rightarrow Q\,\text{IV}$



$\sin 2x = 2 \sin x \cos x$

$= 2 \cdot \frac{-\sqrt{5}}{3} \cdot \frac{2}{3} = \boxed{\frac{-4\sqrt{5}}{9}}$

Oct 8-11:34 AM

Half - Angle:

$\cos 2x = 2\cos^2 x - 1 \Rightarrow \cos^2 x = \frac{1 + \cos 2x}{2}$

$\cos 2x = 1 - 2\sin^2 x \Rightarrow \sin^2 x = \frac{1 - \cos 2x}{2}$

$\sin 15^\circ = \sin(45^\circ - 30^\circ)$

$= \sin 45^\circ \cos 30^\circ - \cos 45^\circ \sin 30^\circ$

$= \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2} \cdot \frac{1}{2} = \frac{\sqrt{6} - \sqrt{2}}{4}$

$15^\circ = \frac{30^\circ}{2}$ $\sin^2 15^\circ = \frac{1 - \cos 2 \cdot 15^\circ}{2}$ $\approx .259$

$= \frac{1 - \cos 30^\circ}{2} = \frac{1 - \frac{\sqrt{3}}{2}}{2}$

$= \frac{2 - \sqrt{3}}{4}$

$\sin 15^\circ = \sqrt{\frac{2 - \sqrt{3}}{4}} = \frac{\sqrt{2 - \sqrt{3}}}{2}$ $\approx .259$

Oct 8-11:38 AM